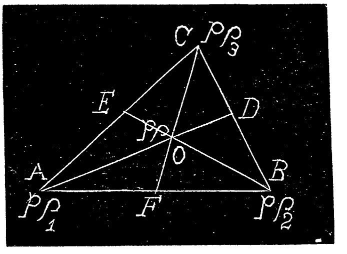
87. We conclude this chapter with an example. *Required to find whether the three medians of a triangle meet in a point.*

Let the vertices of a triangle ABC be located by the unit points $p\rho_1$, $p\rho_2$, $p\rho_3$, and D, E, F be the mid-points of the sides. We have then



$$D=\frac{p\rho_2+p\rho_3}{2},\ E=\frac{p\rho_1+p\rho_3}{2}.$$

If $p\rho$ denote a unit point at O the intersection of AD and BE, and x, y, x', and y' arbitrary scalars, we may write

$$p\rho=xp\rho_1+y\frac{p\rho_2+p\rho_3}{2}=x'p\rho_2+y'\ \frac{p\rho_1+p\rho_3}{2}.$$

Then (20), $x=\frac{1}{2}y'$, $y=y'$; whence $x=\frac{1}{2}y$. But $x+y=1$ (77). Then $x=\frac{1}{3}$, $y=\frac{2}{3}$. Hence

$$p\rho=\tfrac{1}{3}p\rho_1+\tfrac{2}{3}\left(\frac{p\rho_2+p\rho_3}{2}\right)=\tfrac{1}{3}p\rho_1+\tfrac{1}{3}p\rho_2+\tfrac{1}{3}p\rho_3.$$

By symmetry we see that the intersection of AD and CF must be the same point. Or, supposing O to be the intersection of BE and CF, we may test A, O, and D for collinearity directly.

$$\begin{array}{lll} A & \quad O & \quad\quad D \\ \tfrac{1}{3}p\rho_1- & |\ (\tfrac{1}{3}p\rho_1+\tfrac{1}{3}p\rho_2+\tfrac{1}{3}p\rho_3) & +\tfrac{2}{3}(\tfrac{1}{2}(p\rho_2+p\rho_3))\equiv 0. \quad (83). \end{array}$$

It is evident that the above equations can be interpreted as equations of ordinary vector analysis by dropping the p's. In this way is shown the relation existing between point and vector analysis.

[To be Continued.]

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

127. **Proposed by P. S. BERG, A. M., Principal of Schools, Larimore, N. D.**

A man borrows \$1000 of a Building and Loan Association, and at the same time subscribes for 10 \$100-shares of stock. A membership fee of \$1 per share is charged. At the beginning of each month an installment of \$1 per share is paid, also 5% interest and 5% premium on the \$1000. The stock matures in 75 months and the debt is cancelled. What rate of interest does he pay per annum?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

\$10 membership fee, \$10 installment, and \$8.33⅓ interest and premium per month=\$28.33⅓ amount paid down.

\$1000−\$28.33⅓=\$971.66⅔ actual amount received after deducting amounts paid at time of borrowing. On this amount, \$18.33⅓ per month is paid for 74 months.

$\therefore\ 18.33\frac{1}{3}=\dfrac{971.66\frac{2}{3}r(1+r)^{74}}{(1+r)^{74}-1}$, or $18\frac{1}{3}(1+r)^{74}-18\frac{1}{3}=971\frac{2}{3}r(1+r)^{74}$.

$(1+r)^{74}-1=53r(1+r)^{74}$.

$\therefore\ (1-53r)(1+r)^{74}=1$.

$\therefore\ \log(1-53r)+74\log(1+r)=0$.

$\therefore\ r=.0137$, and $12r=.1644=16.44\%$ per annum.

128. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

At what time is the figure 7, on the face of a clock, midway between the hour and minute hands?

Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; D. G. DORRANCE, Jr., Camden, N. Y.; G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.; and the PROPOSER.

Put $7=a$=the given figure on face of clock.

Let x=distance the hour hand travels after a o'clock.

Then $5a-x$=distance the minute hand travels to fulfill the condition between a and $a+1$ o'clock.

$\therefore$ As the minute hand goes 12 times as fast as the hour hand, $5a-x=12x$, and $x=\dfrac{5a}{13}$.

This is the *first* position of the hour hand after a o'clock. For, it will be observed, there are thirteen different positions in all: one after each hour and one at $2a$ o'clock.

For the *second* position, which is between $a+1$ and $a+2$ o'clock, $60+5a-x$=distance the minute hand has to travel. Whence, $60+5a-x=12x$, and $x=\dfrac{60+5a}{13}$.

Now let n represent these 13 positions of the hour hand. Then $x=\dfrac{60(n-1)+5a}{13}$=the nth position of the hour hand after a o'clock.

The time of day for the different positions is, *before* $2a$ o'clock, $5a-\dfrac{60(n-1)+5a}{13}$, or $\dfrac{60(a-n+1)}{13}$ minutes *past* $a+n-1$ o'clock; and, *after* $2a$ o'clock, $\dfrac{60(n-1)+5a}{13}-5a$, or $\dfrac{60(n-1-a)}{13}$ minutes *to* $a+n-1$ o'clock.

Substituting 7 for a, and 1, 2, 3....13, consecutively, for n, we obtain the following thirteen times of day when 7 is midway between the hour and minute hands: $32\frac{4}{13}$ minutes past 7, $27\frac{9}{13}$ minutes past 8, $23\frac{1}{13}$ minutes past 9,